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Performance evaluation of irreversible Stirling and Ericsson heat pump cycles ☆

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Abstract

This communication presents the performance evaluation of irreversible Stirling and Ericsson Heat Pumps Cycles including external and internal irreversibilities along with finite heat capacities of external reservoirs. The external irreversibility is due to finite temperature difference between working fluid and external (source/sink) reservoirs fluid while the internal irreversibilities are due to regenerative heat loss and other entropy generation in the cycle. The heating load is maximized for the given power input. The heating coefficient of performance, the heat transfers to and from the heat pumps and the working fluid temperatures at these conditions have been evaluated. The effect of different parameters (reservoirs temperature, the various effectivenesses and irreversibility parameter), on the performance of these cycles have been studied. It is found that the effect of internal irreversibility parameter is more pronounced than that of other external irreversibility parameters.

Keywords: Internal and external irreversibilities; Stirling and Ericsson heat pumps; Optimum performance; Heating load; Power input; Lagrangian multiplier

1. Introduction

Stirling and Ericsson cycles are one of the important cycle models of refrigerators, airconditioning and heat pump (RAC & HP) systems. These cycle have been utilized by a number of engineering firms in the construction of practical systems, for the production of desirable temperatures. This has promoted the development of new design of these cycles. The concept of finite time thermodynamics was introduced by Curzon and Ahlborn [1] with a novel work on Carnot heat engine in which they remarked that the efficiency of an engine operating at maximum power is given by the formula [$\eta_m = 1 - \sqrt{T_L/T_H}$], which is always smaller than the well-known Carnot formula [$\eta_C = 1 - T_L/T_H$] and agrees much better with the measured efficiencies of operating installations given by them. Wu [2] also applied the similar concept on the finite time Carnot heat engine with finite heat capacity of external reservoirs and shows that the maximum power output and the efficiency at maximum power are function of the inlet temperatures of the external fluids. It is also desirable to have the corresponding results for the coefficient of performance (COP) of the refrigeration, airconditioning and heat pump systems. Leff and Teeters [3] have noted that the straightforwards C–A calculations will not work for reversed Carnot cycle because there is no "Natural Maximum". Blankard [4] has applied the Lagrangian method of undetermined multiplier to find out the COP of endoreversible Carnot heat pump operated at minimum power input for a given heating load. The theory of finite time thermodynamics has been successfully used to investigate the optimal performance of the various cycles viz. endoreversible Carnot, Brayton and Stirling RAC & HP systems for different conditions by Wu [5,6]. In recent years Wu et al. [7–11], Chen [12], Kaushik [13], Kaushik and Kumar [14] and Kumar [15] have also studied the Stirling refrigerators and Stirling & Ericsson heat pumps using finite time thermodynamics and presented the optimum performance parameters as a function of operating conditions.

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Nomenclature

A	area	m^2
C	heat capacitance	$kW \cdot K^{-1}$
C_f	specific heat	$kJ \cdot Kg^{-1} \cdot K^{-1}$
COP	coefficient of performance	
P	power	kW
p	pressure	kPa
Q	heat	kJ
R	heating load	kW
T	temperature	K
t	time	s
U	overall heat transfer coefficient	
v	volume	m^3

Greek symbols

ε	effectiveness
λ_1/λ_2	pressure/volume ratio

Subscript

c	cold side/sink side
h, H	hot side/heat source/heating
L	heat sink
m	minimum operating condition
R	regenerator
s	isentropic/ideal
p	at constant pressure
v	at constant volume
1, 2, 3, 4	state points

In this paper we have presented a more general analysis of Stirling and Ericsson heat pumps with both the external and internal irreversibilities arising due to finite heat capacity of source/sink reservoirs and regenerative losses and the entropy generation within the cycles, respectively. The effects of operating inlet temperatures and capacitance rates of source sink reservoirs, the effectiveness of the regenerator and the internal irreversibility parameter on the heat transfers, COP, the heating load and the power input of the Ericsson and Stirling heat pump cycles have been studied.

2. System description

It is well known that the working substance of Ericsson and Stirling cycles may be a gas, a magnetic material, etc. For different working fluids/substances, these cycles have different performance characteristics. When the working substance of these cycles is an ideal gas, the Stirling cycle consists of two isothermal and two isochoric processes while the Ericsson cycle consists of two isothermal and two isobaric processes as shown on schematic and T-S diagrams in Figs. 1 and 2, respectively. These cycles approximate the expansion stroke of real cycle as an isothermal process 1-2 with irreversible isothermal heat addition at temperature T_c from a heat source of finite heat capacity whose temperature varies from T_{L1} to T_{L2}. The heat addition to the working fluid from the regenerator is modeled as isobaric (in Ericsson cycle) or isochoric (in Stirling cycle) processes 2-3 and 2-3s in real and ideal cycles, respectively. The compression stroke is modeled as an isothermal process 3-4 with irreversible heat rejection at temperature T_h to the heat sink of finite heat capacity whose temperature varies from T_{H1} to T_{H2}. Finally, the heat rejection to the regenerator is modeled as isothermal/isochoric processes 4-1 and 4-1s in real and ideal Ericsson/Stirling heat pump, respectively, completing the cycle.

As mentioned earlier, the heat transfer processes 1-2 and 3-4 in real cycles must occur in finite time. This requires that these heat processes must proceed through a finite temperature difference and therefore, defined as being externally irreversible. Also the entropy change during process 3-4 is more than the entropy change during process 1-2. Thus there is some net entropy generation per cycle and the ratio of entropy change in heat addition (expansion) to that of heat rejection (compression) is called internal irreversibility parameter ($R_{\Delta s}$) which also affects the performance of these cycles. There is also some heat loss through the regenerator, as an ideal regeneration requires infinite regeneration time or area, which is not the case in practice. Hence, it is desirable to consider a real regenerator. By considering all these factors, these heat pumps models become irreversible. The external irreversibility is due to finite temperature difference between the cycle and the external reservoirs and internal

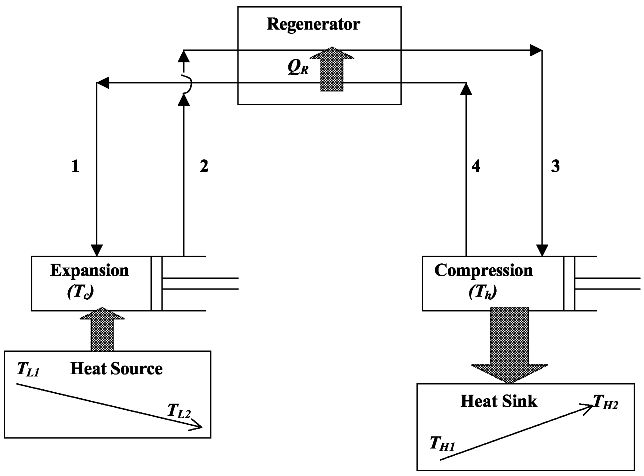

Fig. 1. Schematic of irreversible Stirling/Ericsson heat pump cycles.

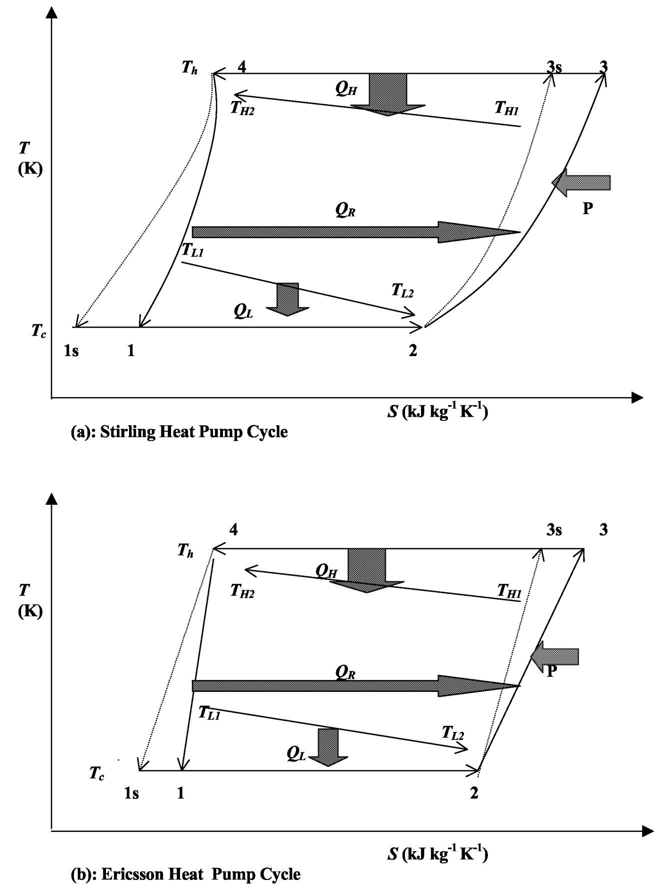

Fig. 2. *T-S* diagrams of irreversible stirling and Ericsson heat pump cycles.

irreversibilities are due to regenerative loss and entropy generation.

3. Thermodynamic analysis

Let Q_c is the amount of heat absorbed from the source at temperature T_c and Q_h is the amount of heat release to the sink at temperature T_h then:

$$Q_h = T_h(S_3 - S_4) = C_H(T_{H2} - T_{H1})t_H \tag{1}$$

$$Q_c = T_c(S_2 - S_1) = C_L(T_{L1} - T_{L2})t_L \tag{2}$$

where $(S_3 - S_4) = nR\ln\lambda_1$ and $(S_2 - S_1) = nR\ln\lambda_2$ where λ_1 & λ_2 are the volume (for Stirling cycle) and pressure (for Ericsson cycle) ratios for the sink and source side in both the cycles, respectively, n is the number of mole for the working fluid and R is the universal gas constant. C_H, C_L and t_H, t_L are the heat capacitance rates of source/sink reservoirs and heat addition/rejection times, respectively. Also from the heat transfer theory the heat Q_h and Q_c will be proportional to the Log Mean Temperature Difference (*LMTD*), i.e.,

$$Q_h = U_H A_H (LMTD)_H t_H \tag{3}$$

$$Q_c = U_L A_L (LMTD)_L t_L \tag{4}$$

where $U_H A_H$ & $U_L A_L$ are the overall heat transfer coefficient-area products and $(LMTD)_H$ & $(LMTD)_L$ are the Log Mean Temperature difference on sink & source side, respectively, and defined as:

$$(LMTD)_H = U_H A_H \left[\frac{(T_h - T_{H1}) - (T_h - T_{H2})}{\ln((T_h - T_{H1})/(T_h - T_{H2}))}\right] \tag{5}$$

$$(LMTD)_L = U_L A_L \left[\frac{(T_{L1} - T_c) - (T_{L2} - T_c)}{\ln((T_{L1} - T_c)/(T_{L2} - T_c))}\right] \tag{6}$$

It is also desirable to consider the finite heat transfer in the regenerative processes. As these cycles in general, do not posses the condition of perfect regeneration. Thus it is assumed reasonably that the regenerator loss per cycle is proportional to the temperature difference of the two processes, i.e.,

$$\Delta Q_R = nC_f(1 - \varepsilon_R)(T_h - T_c) \tag{7}$$

where n is the number of moles, C_f is the specific heat of the working fluid [$C_f = C_v$ for Stirling cycle and $C_f = C_p$ for Ericsson cycle] ε_R is the effectiveness of the regenerator.

Owing, the influence of irreversibility of finite heat transfer, the regenerative time should be finite as compared to the two isothermal processes, as given by earlier workers [8–12] will be:

$$t_R = t_3 + t_4 = 2\alpha(T_h - T_c) \tag{8}$$

where α is the proportionality constant which is independent of the temperature difference but depends on the property of the regenerative material.

Thus, the total cycle time t_{cycle} will be:

$$t_{cycle} = (t_H + t_L + t_R) \tag{9}$$

When all the irreversibilities, mentioned above are taken into account, the net amount of heat released to the sink and absorbed from the source are given by:

$$Q_H = Q_h - \Delta Q_R \tag{10}$$

$$Q_L = Q_c - \Delta Q_R \tag{11}$$

It is well known that the power input, heating load and the coefficient of performance (*COP*) are the important parameters of the heat pumps. Using the above equations we obtain the expression for the power input, heating load and heating *COP* are as follows:

$$P = \frac{Q_H - Q_L}{t_{cycle}} = \frac{Q_h - Q_c}{t_H + t_L + t_R} \tag{12}$$

$$R_H = \frac{Q_H}{t_{cycle}} = \frac{Q_H}{t_H + t_L + t_R} \tag{13}$$

$$COP_H = \frac{R_H}{P} = \frac{Q_H}{Q_h - Q_c} \tag{14}$$

The second law of thermodynamics for irreversible cycle gives:

$$\frac{Q_c}{T_c} - \frac{Q_h}{T_h} < 0 \quad \text{or} \quad \frac{Q_c}{T_c} = R_{\Delta s}\frac{Q_h}{T_h} \tag{15}$$

where $R_{\Delta s}$ is irreversibility parameter and less than unity for real cycle.

Thus from Eqs. (10)–(15), we have:

$$P = \frac{x - R_{\Delta s}}{\frac{x}{k_1(xy - T_{H1})} + \frac{R_{\Delta s}}{k_2(T_{L1} - y)} + b_1(x-1)} \tag{16a}$$

$$R_H = \frac{x - a_1(x-1)}{\frac{x}{k_1(xy - T_{H1})} + \frac{R_{\Delta s}}{k_2(T_{L1} - y)} + b_1(x-1)} \tag{16b}$$

$$COP_H = \frac{x - a_1(x-1)}{x - R_{\Delta s}} \tag{16c}$$

where $k_1 = \varepsilon_H C_H$, $k_2 = \varepsilon_L C_L$, $x = T_h/T_c$, $y = T_c$, $b_1 = 2/(\alpha n R \ln\lambda_1)$ and $a_1 = C_f(1-\varepsilon_R)/R\ln\lambda_1$.

The purpose of any heat pump is to reject as much heat as possible to the sink (space to be heated) with the expenditure of as little work as possible. This implies that we should do our best to minimize the power input for a given heating load or maximize the heating load for a given power input. For this end, we introduce the Lagrangian

$$L = R_H + \lambda P = \frac{[x - a_1(x-1)] + \lambda(x - R_{\Delta s})}{\frac{x}{k_1(xy - T_{H1})} + \frac{R_{\Delta s}}{k_2(T_{L1} - y)} + b_1(x-1)} \tag{17}$$

where λ is the Lagrangian multiplier. From Euler–Lagrange equation:

$$\frac{\partial L}{\partial y} = 0 \tag{18}$$

and Eq. (18) we find the optimal relation:

$$x(T_{L1} - y) = \sqrt{\frac{k_1 R_{\Delta s}}{k_2}}(xy - T_{H1}) \tag{19}$$

Substituting Eq. (19) into Eqs. 16(a)–(c) gives:

$$P = \frac{x - R_{\Delta s}}{((x + R_{\Delta s})k_4)/(xT_{L1} - T_{H1}) + b_1(x-1)} \tag{20a}$$

$$R_H = \frac{[x - a_1(x-1)]}{((x + R_{\Delta s})k_4)/(xT_{L1} - T_{H1}) + b_1(x-1)} \tag{20b}$$

$$COP_H = \frac{x - a_1(x-1)}{x - R_{\Delta s}} \tag{20c}$$

where

$$k_4 = (1 + k_3)\left[\frac{1}{k_1} + \frac{1}{k_2 k_3}\right] \quad \text{and} \quad k_3 = \sqrt{\frac{k_1 R_{\Delta s}}{k_2}} \tag{21}$$

Eq. (20b) shows that heating load is not monotonically increasing function of power input and there exists a maximum value of heating load. Thus maximizing R_H with respect x, yields

$$x_{opt} = -\frac{B \pm \sqrt{B^2 - 4AC}}{2A} \tag{22}$$

where

$$A = k_4(1 - a_1)T_{L1} - b_1 T_{L1}^2$$

$$B = 2\left[k_4(1 - a_1)R_{\Delta s}T_{L1} + b_1 T_{H1}T_{L1}\right] \quad \text{and}$$

$$C = k_4\left[a_1 T_{H1} + a_1 R_{\Delta s}T_{L1} - (1 - a_1)R_{\Delta s}T_{H1}\right] - b_1 T_{H1}^2$$

Substituting Eq. (22) into Eqs. 20(a)–(c) we find the optimal values of the power input, heating load and heating coefficient of performance of these heat pumps.

4. Special cases

(1) When $\varepsilon_R = 1.00$, i.e., these cycles posses the condition of perfect regeneration. The performance of these cycles obtained is also given by Eqs. 20(a)–(c) by substituting $a_1 = 0.0$. Thus, the performance characteristics of Ericsson and Stirling heat pump cycles with perfect regeneration are equivalent to that of similar Carnot heat pump operating at the same condition in which the time of two adiabatic processes is given by Eq. (8). Although physically for finite regeneration time, ε_R should be less than unity. This shows that in the investigation of these cycles it would be impossible to obtain new conclusion if the regenerative losses were not considered.

(2) When $R_{\Delta s} = 1.00$, i.e., these cycles are internally reversible. The performances are still given by Eqs. 20(a)–(c) in which $R_{\Delta s} = 1.00$.
The performance of these cycles is equivalent to that of an endoreversible Carnot heat pump for the same operating conditions.

(3) When $t_R = \gamma(t_H + t_L)$, i.e., the time of regeneration is proportional to that of two isothermal processes. The performance characteristics are given by:

$$P = \frac{(x_1 - R_{\Delta s})(x_1 T_{L1} - T_{H1})}{(1+\gamma)x_1 k_4} \tag{23a}$$

$$R_H = \frac{[x_1 - a_1(x_1 - 1)](x_1 T_{L1} - T_{H1})}{(1+\gamma)x_1 k_4} \tag{23b}$$

$$COP_H = \frac{x_1 - a_1(x_1 - 1)}{x_1 - R_{\Delta s}} \tag{23c}$$

where

$$x_1 = -B_1 \pm \sqrt{\frac{B_1^2 - 4A_1 C_1}{2A_1}}$$

and

$$A_1 = (1 - a_1)T_{L1}, \qquad B_1 = 2R_{\Delta s}(1 - a_1)T_{L1} \quad \text{and}$$

$$C_1 = a_1\left[R_{\Delta s}(T_{L1} - T_{H1}) + T_{H1}(1 + R_{\Delta s})\right]$$

5. Discussion of results

In order to have the numerical appreciation of the results, we have considered the heat sink and heat source temperatures $T_{H1} = 310$–335 K and $T_{L1} = 270$–295 K, respectively, the effectiveness of the heat exchangers (ε_H, ε_L and ε_R) in the range 0.40–1.00, the volume ratio $v_2/v_1 = 2.50$, the pressure ratio $p_1/p_2 = 2.64$ and the irreversibility parameter $R_{\Delta s}$ in the range 0.50–1.00. We have studied the effect of each of these parameters (while keeping the others as constant) on the power input, the heating load and the heating *COP* of both the heat pumps and the results are given below:

Tables 1–3 show that the effect of the various heat exchangers ($\varepsilon_H, \varepsilon_L$ and ε_R) on the heat transfer to and from the heat pumps, the regenerative heat transfer, the power

Table 1
Effect of ε_H on the performance of Stirling and Ericsson heat pumps ($\varepsilon_L = 0.75$, $\varepsilon_R = 0.90$, $T_{H1} = 330$ K, $T_{L1} = 290$ K, $C_H = C_L = 1.00$ and $R_{\Delta s} = 0.80$)

	Stirling cycle								Ericsson cycle							
ε_H	Q_H	Q_L	Q_R	P	R_H	COP	T_h	T_c	Q_H	Q_L	Q_R	P	R_H	COP	T_h	T_c
–	kJ	kJ	kJ	kW	kW	–	K	K	kJ	kJ	kJ	kW	kW	–	K	K
0.40	65.05	49.58	50.13	0.90	1.45	1.61	354.99	277.25	92.28	67.11	81.58	1.23	2.10	1.70	363.76	273.43
0.50	64.67	49.84	48.04	0.96	1.60	1.67	351.90	277.39	91.52	67.67	77.32	1.30	2.30	1.77	359.30	273.70
0.60	64.40	50.02	46.61	0.99	1.70	1.71	349.75	277.47	90.99	68.03	74.43	1.35	2.45	1.82	356.26	273.85
0.70	64.20	50.14	45.58	1.02	1.79	1.75	348.17	277.49	90.60	68.27	72.38	1.38	2.57	1.86	354.06	273.92
0.75	64.12	50.18	45.17	1.04	1.82	1.76	347.53	277.48	90.45	68.37	71.57	1.40	2.62	1.87	353.17	273.93
0.80	64.04	50.22	44.81	1.05	1.85	1.77	346.96	277.47	90.31	68.44	70.87	1.41	2.66	1.88	352.39	273.92
0.85	63.98	50.25	44.50	1.06	1.88	1.78	346.46	277.45	90.18	68.51	70.26	1.42	2.70	1.89	351.70	273.91
0.90	63.92	50.27	44.23	1.06	1.90	1.79	346.01	277.42	90.07	68.56	69.72	1.43	2.73	1.90	351.09	273.88
0.95	63.86	50.29	43.99	1.07	1.93	1.80	345.61	277.39	89.97	68.60	69.25	1.44	2.76	1.91	350.53	275.85
1.00	63.82	50.31	43.78	1.08	1.94	1.80	345.24	277.35	89.88	68.64	68.84	1.45	2.79	1.92	350.04	275.81

Table 2
Effect of ε_L on the performance of Stirling and Ericsson heat pumps ($\varepsilon_H = 0.75$, $\varepsilon_R = 0.90$, $T_{H1} = 330$ K, $T_{L1} = 290$ K, $C_H = C_L = 1.00$ and $R_{\Delta s} = 0.80$)

	Stirling cycle								Ericsson cycle							
ε_L	Q_H	Q_L	Q_R	P	R_H	COP	T_h	T_c	Q_H	Q_L	Q_R	P	R_H	COP	T_h	T_c
–	kJ	kJ	kJ	kW	kW	–	K	K	kJ	kJ	kJ	kW	kW	–	K	K
0.40	63.79	49.06	47.75	0.94	1.56	1.66	347.34	273.29	89.90	66.25	76.66	1.23	2.26	1.76	353.25	268.37
0.50	63.91	49.54	46.58	0.98	1.67	1.70	347.26	275.03	90.09	67.17	74.33	1.33	2.41	1.81	352.99	270.69
0.60	64.00	49.86	45.83	1.01	1.75	1.73	347.31	276.23	90.24	67.77	72.87	1.37	2.51	1.84	352.97	272.28
0.70	64.08	50.09	45.35	1.03	1.80	1.75	347.44	277.12	90.38	68.20	71.92	1.39	2.59	1.86	353.08	273.45
0.75	64.12	50.18	45.17	1.04	1.82	1.76	347.53	277.48	90.45	68.37	71.57	1.40	2.62	1.87	353.17	273.93
0.80	64.15	50.26	45.02	1.04	1.84	1.76	347.62	277.80	90.51	68.51	71.29	1.41	2.64	1.88	353.28	274.34
0.85	64.18	50.33	44.91	1.05	1.85	1.77	347.73	278.09	90.57	68.64	71.06	1.41	2.66	1.88	353.40	274.72
0.90	64.22	50.39	44.81	1.05	1.87	1.77	347.84	278.35	90.62	68.76	70.88	1.42	2.68	1.89	353.53	275.05
0.95	64.25	50.45	44.74	1.15	1.87	1.78	347.95	278.58	90.68	68.86	70.74	1.42	2.69	1.89	353.67	275.35
1.00	63.28	50.50	44.68	1.06	1.88	1.78	348.07	278.79	90.73	68.95	70.62	1.42	2.70	1.90	353.62	275.93

Table 3
Effect of ε_R on the performance of Stirling and Ericsson heat pumps ($\varepsilon_H = \varepsilon_L = 0.75$, $T_{H1} = 330$ K, $T_{L1} = 290$ K, $C_H = C_L = 1.00$ and $R_{\Delta s} = 0.80$)

	Stirling cycle								Ericsson cycle							
ε_R	Q_H	Q_L	Q_R	P	R_H	COP	T_h	T_c	Q_H	Q_L	Q_R	P	R_H	COP	T_h	T_c
–	kJ	kJ	kJ	kW	kW	–	K	K	kJ	kJ	kJ	kW	kW	–	K	K
0.60	53.58	45.15	18.44	0.18	0.04	0.21	331.62	288.74	73.13	58.83	30.91	0.64	0.42	0.66	336.43	285.11
0.70	56.06	46.19	24.89	0.52	0.32	0.62	335.46	285.84	77.26	60.51	41.24	0.93	0.92	0.98	340.75	282.04
0.75	57.57	46.86	28.94	0.68	0.58	0.86	337.91	284.06	79.83	62.25	47.50	1.07	1.62	1.17	343.36	280.25
0.80	59.37	47.70	33.63	0.82	0.91	1.11	340.73	282.06	82.83	63.88	54.60	1.20	1.65	1.38	346.30	278.29
0.85	61.53	48.79	39.03	0.94	1.33	1.42	343.94	279.86	86.35	65.90	62.60	1.31	2.11	1.61	349.57	276.17
0.90	64.12	50.18	45.17	1.04	1.82	1.76	347.53	277.48	90.45	68.37	71.57	1.40	2.62	1.87	353.17	273.93
0.95	67.18	51.96	52.08	1.11	2.38	2.15	351.48	274.97	95.20	71.37	81.55	1.47	3.17	2.16	357.10	271.56
1.00	70.00	54.65	55.30	2.66	3.25	2.65	351.89	274.71	99.33	75.76	84.90	1.61	4.12	2.57	356.52	270.91

input, the heating load, the heating *COP* and the working fluid temperatures of both the heat pumps.

Effect of ε_H. It can be seen from Table 1 that as the effectiveness of the sink side heat exchanger increases, the heat transfer to the heat pumps, the power input, the heating load, and the heating *COP* increase while the heat transfer from the heat pump, the regenerative heat transfer and the sink side working fluid temperature decrease. The effect of ε_H is more pronounced for heating load and less pronounced for the source side working fluid temperature.

Effect of ε_L. Table 2 shows that as effectiveness of the source side heat exchanger increases, the heat transfer to the heat pumps, the power input, the heating load, the heating *COP* and the working fluid temperatures increase while the heat transfer from the heat pumps and the regenerative heat transfer decrease. The effect of the ε_L is more pronounced for the heating load and less pronounced for the sink side working fluid temperature.

Effect of ε_R. Table 3 shows that as the effectiveness of the regenerator increases, the heat transfer to and from the heat pumps, the regenerative heat transfer, the

Table 4
Effect of T_{H1} on the performance of Stirling and Ericsson heat pumps ($\varepsilon_H = \varepsilon_L = 0.75$, $\varepsilon_R = 0.90$, $T_{L1} = 290$ K, $C_H = C_L = 1.00$ and $R_{\Delta s} = 0.80$)

	Stirling cycle								Ericsson cycle							
T_{H1} K	Q_H kJ	Q_L kJ	Q_R kJ	P kW	R_H kW	COP –	T_h K	T_c K	Q_H kJ	Q_L kJ	Q_R kJ	P kW	R_H kW	COP –	T_h K	T_c K
310	61.53	51.39	32.86	1.08	2.38	2.21	327.64	276.68	86.74	70.18	53.68	1.42	3.19	2.24	332.74	273.30
313	61.92	51.21	34.71	1.07	2.28	2.14	330.62	276.80	87.30	69.91	56.37	1.42	3.09	2.18	335.80	273.39
315	62.18	51.09	35.94	1.07	2.22	2.09	332.62	276.88	87.67	69.73	58.16	1.42	3.03	2.14	337.85	273.46
317	62.43	50.97	37.17	1.06	2.16	2.04	334.61	276.96	88.04	69.55	59.94	1.41	2.97	2.10	339.89	273.52
320	62.82	50.79	39.02	1.05	2.07	1.97	337.59	277.09	88.59	69.27	62.63	1.41	2.88	2.04	342.96	273.62
322	63.08	50.67	40.25	1.05	2.02	1.92	339.58	277.17	88.97	69.09	64.42	1.41	2.82	2.01	345.00	273.68
324	63.34	50.54	41.48	1.05	1.97	1.88	341.87	277.25	89.34	68.91	66.20	1.41	2.77	1.97	347.04	273.74
326	63.60	50.42	42.71	1.04	1.92	1.84	343.55	277.33	89.71	68.73	67.99	1.40	2.72	1.94	349.09	273.80
328	63.86	50.30	43.94	1.04	1.87	1.80	345.54	277.40	90.08	68.55	69.78	1.40	2.66	1.90	351.13	273.86
330	64.12	50.18	45.17	1.04	1.82	1.76	347.33	277.48	90.45	68.37	71.57	1.40	2.62	1.87	353.17	273.93
332	64.37	50.06	46.40	1.03	1.78	1.72	349.51	277.56	90.82	69.19	73.36	1.40	2.57	1.84	355.21	273.99
335	64.76	49.88	48.24	1.03	1.71	1.67	352.49	277.88	91.37	67.91	76.04	1.39	2.50	1.79	358.27	274.08

Table 5
Effect of T_{L1} on the performance of Stirling and Ericsson heat pumps ($\varepsilon_H = \varepsilon_L = 0.75$, $\varepsilon_R = 0.90$, $T_{H1} = 330$ K, $C_H = C_L = 1.00$ and $R_{\Delta s} = 0.80$)

	Stirling cycle								Ericsson cycle							
T_{L1} K	Q_H kJ	Q_L kJ	Q_R kJ	P kW	R_H kW	COP –	T_h K	T_c K	Q_H kJ	Q_L kJ	Q_R kJ	P kW	R_H kW	COP –	T_h K	T_c K
270	62.81	45.72	55.42	0.99	1.37	1.38	346.70	260.76	88.70	62.04	86.41	1.36	2.12	1.56	352.81	257.13
272	62.96	46.21	54.28	0.99	1.41	1.42	346.80	262.62	88.89	62.74	84.77	1.36	2.17	1.59	352.88	259.00
274	63.10	46.71	53.15	1.00	1.46	1.46	346.90	264.48	89.09	63.44	83.12	1.37	2.22	1.62	352.90	260.86
276	63.25	47.20	52.01	1.00	1.51	1.50	346.99	266.34	89.28	64.15	81.47	1.37	2.27	1.66	352.94	262.73
278	63.39	47.70	50.87	1.01	1.56	1.54	347.08	268.19	89.47	64.85	79.82	1.38	2.32	1.69	352.98	264.59
280	63.54	48.20	49.73	1.01	1.61	1.58	347.18	270.08	89.67	65.55	78.17	1.38	2.38	1.72	353.02	266.46
282	63.68	48.69	48.59	1.02	1.66	1.63	347.27	271.91	89.86	66.26	76.52	1.38	2.44	1.76	353.06	270.19
284	63.83	49.19	47.45	1.03	1.71	1.67	347.35	273.77	90.06	66.96	74.87	1.39	2.49	1.80	353.09	272.06
286	63.97	49.69	46.31	1.03	1.77	1.71	347.44	275.62	90.25	67.66	73.22	1.39	2.55	1.83	353.13	273.96
288	64.12	50.18	45.17	1.04	1.82	1.76	347.53	277.48	90.45	68.37	71.57	1.40	2.62	1.87	353.17	273.93
290	64.33	50.93	43.45	1.05	1.91	1.83	347.65	280.27	90.74	69.42	69.09	1.41	2.71	1.93	353.23	276.73
295	64.48	51.42	42.31	1.05	1.97	1.88	347.73	282.13	90.93	70.13	67.44	1.41	2.78	1.97	353.26	278.59

power input, the heating load, the heating *COP* and the sink side working fluid temperature increase while the source side working fluid temperature decreases. The effect of ε_R is more pronounced for the heating load and less pronounced for the source side working fluid temperature.

Tables 4 and 5 show the effect of source and sink inlet temperatures (T_{L1} & T_{H1}) on the heat transfers, the power input, the heating load, the heating *COP* and the working fluid temperatures of these heat pumps.

Effect of T_{H1}. It can be seen from Table 4 that as the inlet temperature of the sink fluid increases, the heat transfer from the heat pumps, the regenerative heat transfer and the working fluid temperature increase while the heat transfer to the heat pumps, the power input, the heating load and the heating *COP* decrease. The effect of T_{H1} is more pronounced for the regenerative heat transfer and less pronounced for the source side working fluid temperature. Thus, it is desirable to have lower T_{H1} for better performance of both the heat pumps.

Effect of T_{L1}. Table 5 shows that as T_{L1} increases, the heat transfer to and from the heat pumps, the power input, the heating load, the heating *COP* and the working fluid temperatures increase while the regenerative heat transfer decreases. The effect of the T_{L1} is more pronounced for the heating load and less pronounced for the sink side working fluid temperature. Thus, it is desirable to have higher T_{L1} for better performance of both the heat pumps.

Tables 6–8 show the effect of heat capacitance rates (C_H & C_L) and the internal irreversibility parameter ($R_{\Delta s}$) on the heat transfers, the power input, the heating load, the heating *COP* and the working fluid temperatures of these heat pumps.

Effect of C_H. Table 6 shows that as the heat capacitance rates of the sink side fluid increases, the heat transfers (Q_H & Q_R) and the sink side working

Table 6
Effect of C_H on the performance of Stirling and Ericsson heat pumps ($\varepsilon_H = \varepsilon_L = 0.75$, $\varepsilon_R = 0.90$, $T_{H1} = 330$ K, $T_{L1} = 290$ K, $C_L = 1.00$ and $R_{\Delta s} = 0.80$)

	Stirling cycle								Ericsson cycle							
C_H KW·K^{-1}	Q_H kJ	Q_L kJ	Q_R kJ	P kW	R_H kW	COP –	T_h K	T_c K	Q_H kJ	Q_L kJ	Q_R kJ	P kW	R_H kW	COP –	T_h K	T_c K
0.50	65.17	49.50	50.81	0.89	1.41	1.59	355.99	277.20	92.53	66.93	82.98	1.21	2.04	1.68	365.21	273.33
0.60	64.84	49.73	48.98	0.93	1.53	1.64	353.29	277.33	91.86	67.42	79.23	1.27	2.21	1.74	361.30	273.58
0.70	64.59	49.89	47.64	0.97	1.63	1.68	351.29	277.42	91.37	67.77	76.49	1.31	2.34	1.79	358.44	273.75
0.80	64.40	50.02	46.61	0.99	1.70	1.71	349.75	277.47	90.99	68.03	74.43	1.35	2.45	1.82	356.26	273.85
0.90	64.24	50.11	45.81	1.02	1.77	1.74	348.52	277.49	90.69	68.22	72.83	1.37	2.54	1.85	354.55	273.91
1.00	64.12	50.18	45.17	1.04	1.82	1.76	347.53	277.48	90.45	68.37	71.57	1.40	2.62	1.87	353.17	273.93
1.10	64.01	50.23	44.65	1.05	1.87	1.77	346.70	277.46	90.24	68.48	70.55	1.42	2.68	1.89	352.04	273.92
1.20	63.92	50.27	44.23	1.06	1.90	1.79	346.01	277.42	90.07	68.56	69.72	1.43	2.73	1.90	351.09	273.88
1.30	63.84	50.30	43.88	1.08	1.94	1.80	345.42	277.37	89.92	68.62	69.04	1.45	2.77	1.92	350.28	273.83
1.40	64.77	50.32	43.59	1.08	1.96	1.81	344.91	277.31	89.79	68.67	68.47	1.46	2.81	1.93	349.58	273.77
1.50	63.71	50.34	43.34	1.09	1.98	1.82	344.46	277.25	89.68	68.70	68.00	1.47	2.84	1.94	348.98	273.69

Table 7
Effect of C_L on the performance of Stirling and Ericsson heat pumps ($\varepsilon_H = \varepsilon_L = 0.75$, $\varepsilon_R = 0.90$, $T_{H1} = 330$ K, $T_{L1} = 290$ K, $C_H = 1.00$ and $R_{\Delta s} = 0.80$)

	Stirling cycle								Ericsson cycle							
C_L KW·K^{-1}	Q_H kJ	Q_L kJ	Q_R kJ	P kW	R_H kW	COP –	T_h K	T_c K	Q_H kJ	Q_L kJ	Q_R kJ	P kW	R_H kW	COP –	T_h K	T_c K
0.50	63.76	48.91	48.15	0.93	1.53	1.64	347.39	272.73	89.85	65.95	77.45	1.27	2.21	1.74	353.37	267.61
0.60	63.85	49.32	47.10	0.97	1.62	1.68	347.28	274.24	90.00	66.75	75.36	1.31	2.34	1.79	353.08	269.64
0.70	63.93	49.63	46.36	0.99	1.70	1.71	347.26	275.37	90.13	67.33	73.90	1.34	2.44	1.82	352.97	271.14
0.80	64.00	49.86	45.83	1.01	1.75	1.73	347.31	276.23	90.24	67.77	72.87	1.37	2.51	1.84	352.97	272.28
0.90	64.06	50.04	45.45	1.03	1.79	1.75	347.40	276.97	90.35	68.10	72.12	1.38	2.57	1.86	353.04	273.19
1.00	64.12	50.18	45.17	1.04	1.82	1.76	347.53	277.48	90.45	68.37	71.57	1.40	2.62	1.87	353.17	273.93
1.10	64.17	50.30	44.96	1.04	1.85	1.77	347.67	277.95	90.54	68.58	71.17	1.41	2.65	1.88	353.34	274.54
1.20	64.22	50.39	44.81	1.05	1.87	1.77	347.84	278.35	90.62	68.76	70.88	1.42	2.68	1.89	353.35	275.05
1.30	64.26	50.47	44.70	1.06	1.88	1.78	348.01	278.69	90.71	68.90	70.68	1.42	2.70	1.89	353.75	275.49
1.40	64.31	50.54	44.63	1.06	1.89	1.78	348.19	278.98	90.78	69.03	70.53	1.43	2.71	1.90	353.97	275.88
1.50	64.35	50.60	44.58	1.06	1.90	1.79	348.35	279.24	90.86	69.13	70.44	1.43	2.72	1.90	354.21	276.21

Table 8
Effect of $R_{\Delta s}$ on the performance of Stirling and Ericsson heat pumps ($\varepsilon_H = \varepsilon_L = 0.75$, $\varepsilon_R = 0.90$, $T_{H1} = 330$ K, $T_{L1} = 290$ K and $C_H = C_L = 1.00$)

	Stirling cycle								Ericsson cycle							
$R_{\Delta s}$ –	Q_H kJ	Q_L kJ	Q_R kJ	P kW	R_H kW	COP –	T_h K	T_c K	Q_H kJ	Q_L kJ	Q_R kJ	P kW	R_H kW	COP –	T_h K	T_c K
0.50	64.39	50.65	44.56	1.78	1.90	1.07	348.57	279.47	90.93	69.22	70.39	2.33	2.73	1.17	354.45	276.51
0.60	64.28	50.50	44.68	1.53	1.88	1.23	348.07	278.79	90.73	68.95	70.62	2.01	2.70	1.34	353.82	275.63
0.70	64.19	50.34	44.89	1.28	1.86	1.45	347.74	278.13	90.57	68.66	71.03	1.70	2.66	1.56	353.42	274.77
0.75	64.15	50.26	45.02	1.16	1.84	1.59	347.62	277.80	90.51	68.51	71.29	1.55	2.64	1.70	353.28	274.34
0.80	64.12	50.18	45.17	1.04	1.82	1.76	347.53	277.48	90.45	68.37	71.57	1.40	2.62	1.87	353.17	273.93
0.90	64.08	50.10	45.32	0.92	1.80	1.97	347.45	277.17	90.39	68.22	71.87	1.25	2.59	2.07	353.09	273.51
0.95	64.05	50.02	45.49	0.80	1.79	2.24	347.39	276.85	90.34	68.07	72.19	1.10	2.57	2.33	353.03	273.10
1.00	64.03	49.94	45.66	0.68	1.77	2.59	347.34	276.54	90.29	67.92	72.52	0.96	2.54	2.65	352.99	272.69

fluid temperature decrease but the power input, the heating load and the heating *COP* increase while the source side working fluid temperature initially increases and then decreases The effect of C_H is more pronounced for the heating load and less pronounced for the source side working fluid temperature.

Effect of C_L. Table 7 shows that as the heat capacitance rates of the source side fluid increases, the heat transfers to and from the heat pumps (Q_H & Q_L), the power input, heating load the heating *COP* and the working fluid temperatures increase while the regenerative heat transfer (Q_R) decreases. The effect of C_L is more pronounce for the heating load and less pronounced for the sink side working fluid temperature.

Since, higher value of C_H decreased the heat transfers and increases the performance while higher value of C_L increased both the heat transfers and the performance thus it is desirable to have higher C_H rather than higher C_L.

Effect of $R_{\Delta s}$. Table 8 shows that as the internal irreversibility parameter increases, the heat transfers to and from the heat pump, the power input, the heating load and the working fluid temperatures decrease while the regenerative heat transfer and the heating *COP* increase. The effect of $R_{\Delta s}$ is more pronounced for the power input and less pronounced for the sink side working fluid temperature. Thus, it is desirable to have higher $R_{\Delta s}$ for better performance of the both cycles. It can also be seen that the effect of $R_{\Delta s}$ is more pronounced than the other parameters on the performance of both the heat pump cycles.

6. Conclusions

The performance characteristics of irreversible Ericsson and Stirling heat pump cycles have been evaluated including external as well as internal irreversibilities. The external irreversibility is due to finite temperature difference and internal irreversibilities are due to regenerative loss and entropy generation in the cycle. It is found that the effectiveness of each heat exchanger, the source side external inlet temperature of the fluids, the heat capacitance rate on sink side external fluid and the irreversibility parameter should be higher enough while the sink side inlet temperature of the fluids and the heat capacitance rates on source side should be lower for better performance of both the heat pumps. Since, the larger heat capacitance allows the fluid to reject the heat at lower temperature thus it is desirable to have higher capacitance rate on sink side in comparison to that on source side (i.e., $C_H > C_L$) for better performance of both the cycles. Hence, the present analysis will be useful and more general for evaluating the performance of these cycles and other regenerative cycles as well.

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